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JEE MAIN

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JEE MAIN 2017 COMPLETE ANSWER KEY WITH FULL SOLUTION

CODE
A

Important Instructions :

1. Immediately fill in the particulars on this page of the Test Booklet with only Black Ball Point Pen provided in the examination ball.
2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.
3. The test is of 3 hours duration.
4. The Test Booklet consists of 90 questions. The maximum marks are 360.
5. There are three parts in the question paper A, B, C consisting of Phsics, Chemistry and Mathematics having 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for correct response.
6. Candidates will be awarded marks as stated above in instruction No. 5 for correct response of each question ¼ (one-fourth) marks of the total marks allotted to the question (i.e. 1 mark) will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
7. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deucted accordingly as per instruction 6 above.
8. For writing particulars / marking responses on Side-1 and Side-2 of the Answer Sheet use only Black Ball Point Pen provided in the examination hall.
9. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. except the Admit Card inside the examination room / hall.
10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in four pages (page 20-23) at the end of the booklet.
11. On completion of the best, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room / Hall. However, the candidates are allowed to take away this Test Booklet with them.
12. The CODE for this Booklet is A. Make sure that the CODE printed on Side-2 of the Answer Sheet and also tally the serial number of the Test Booklet and Answer Sheet are the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.
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Part A – Physics

All The Graphs / Diagrams Given Are Schematic And Not Drawn to Scale

1. A man grows into a giant such that his density remains same, the stress in the leg will change by a factor of :

(1) 9 (2) $\frac{1}{9}$

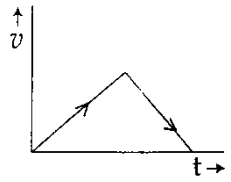
(3) 81 (4) $\frac{1}{81}$

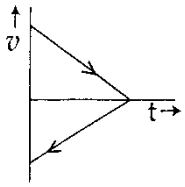
Ans. (1)

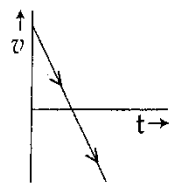
Stress = F/A = $\frac{Al\rho g}{A}$

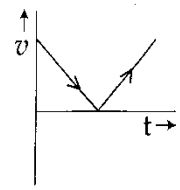
$\Rightarrow$ stress α length

2. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time ?

(1) 

(2) 

(3) 

(4) 

Ans. (3)

Conceptual.

3. A body of mass m = 10^{-2} kg is moving in a medium and experiences a frictional force F = $-kv^2$. Its initial speed is v_0 = 10 ms^{-1}. If after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be

(1) 10^{-3} kg m^{-1} (2) 10^{-3} kg s^{-1}

(3) 10^{-4} kg m^{-1} (4) 10^{-1} kg m^{-1} s^{-1}

Ans. (3)

After 10 sec v = 5 m/s

$$a = -\frac{kv^2}{m} \Rightarrow \frac{dv}{dt} = -\frac{dv^2}{m}$$

$$\Rightarrow \int_{10}^{5}\frac{dv}{v} = -\frac{k}{m}\int_{0}^{10}dt$$

$$\Rightarrow \int_{10}^{5}\frac{dv}{v} = -\frac{k}{m}\int_{0}^{10}dt$$

4. A time dependent force F = 6t acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 sec. will be

(1) 4.5 J (2) 22 J

(3) 9 J (4) 18 J

Ans. (1)

F = 6t

$$\Rightarrow m\frac{dv}{dt} = 6t$$

$$\Rightarrow \frac{dv}{dt} = 6t$$

$$\int_0^{v_0} dv = \int_0^1 6t$$

$v_0 = 3 m/s$

$\Delta w = \Delta KE = 4.5$ J

5. The moment of inertia of a uniform cylinder of length l and radius R about its perpendicular bisector is 1. What is the ratio I/R such that the moment of inertia is minimum ?

(1) $\sqrt{\frac{3}{2}}$ (2) $\frac{\sqrt{3}}{2}$

(3) 1 (4) $\frac{3}{\sqrt{2}}$

Ans. (1)

$$I = \frac{ml^2}{12} + \frac{mR^2}{4}$$

$v = \pi R^2 l$

$$l = \frac{v}{\pi R^2}$$

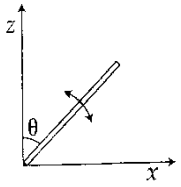
$$I = \frac{m}{12} \times \frac{v^2}{\pi^2 R^4} + \frac{mR^2}{4}$$

$$\frac{dI}{dR} = \frac{mv^2}{12\pi^2} \frac{-4}{RJ} + \frac{1}{4} m^2 R = 0$$

$$\frac{4mv^2}{12\pi^2 RJ} = \frac{1}{4} h \times 2R$$

$$\Rightarrow \frac{l}{R} = \sqrt{\frac{3}{2}}$$

6. A slender uniform rod of mass M and length l is provided at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is :



(1) $\frac{3g}{2l} \sin\theta$ (2) $\frac{2g}{3l} \sin\theta$

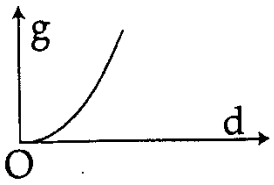
(3) $\frac{3g}{2l} \cos\theta$ (4) $\frac{2g}{3l} \cos\theta$

Ans. (1)

Conceptual

7. The variation of acceleration to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius) :

(1)



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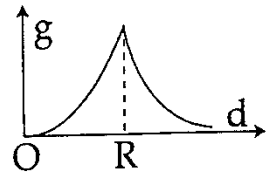

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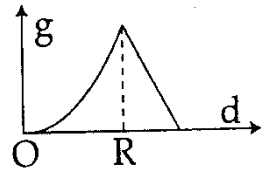

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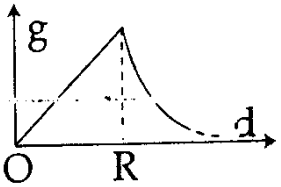


Ans. (4)

Conceptual

8. A copper ball of mass 100 gm is at a temperature T. It is dropped in a copper calorimeter of mass 100 gm, filled with 170 gm of water at room temperature. Subsequently, the temperature of the system is found to be 75°C. T is given by: (Given : room temperature = 30° C, specific heat of copper = 0.1 cal/gm°C)

(1) 800°C (2) 885°C

(3) 1250°C (4) 825°C

Ans. (2)

Heat loss = Heat gain

$\Rightarrow$ T = 885° C

9. An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by :

(1) $\frac{P}{3\alpha K}$ (2) $\frac{P}{\alpha K}$

(3) $\frac{3\alpha}{PK}$ (4) $3PK\alpha$

Ans. (1)

$$K = \frac{\Delta P}{\frac{\Delta V}{V}} \Rightarrow \frac{\Delta V}{V} = \frac{\Delta P}{K}$$

$$3\alpha\Delta T = \frac{P}{K}$$

$$\Delta T = \frac{P}{3\alpha K}$$

10. C_P and C_v are specific heat at constant pressure and constant volume respectively. It is observed that $C_p - C_v$ = a for hydrogen gas ; $C_p - C_v$ = b for nitrogen gas. The correct relation between a and b is :

(1) $a = \frac{1}{14}b$ (2) a = b

(3) a = 14 b (4) a = 28 b

Ans. (3)

Here C_p & C_v are hot molar specific heat.

11. The temperature of an open room of volume 30 m^3 increases from 17°C to 27°C due to the sunshine. The atmospheric pressure in the room remains 1×10^5 Pa. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be

(1) -1.61×10^{23} (2) 1.38×10^{23}

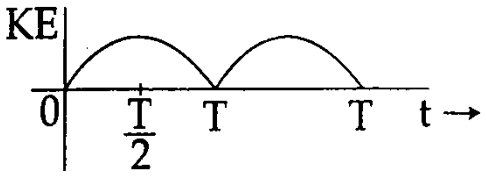
(3) 2.5×10^{25} (4) -2.5×10^{25}

Ans. (4)

PV = nRT

12. A particle is executing simple harmonic motion with a time period T. At time t = 0, it is at its position of equilibrium. The kinetic energy – time graph of the particle will look like :

(1)



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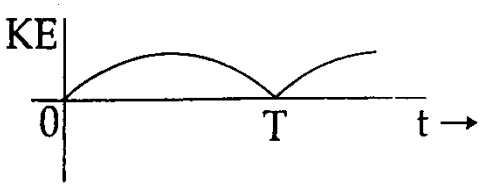

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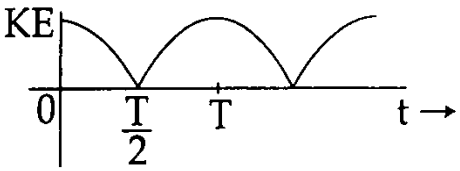

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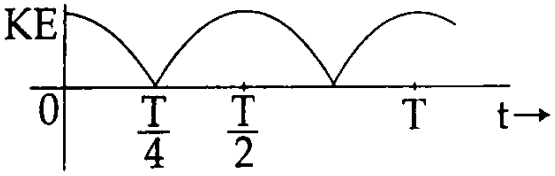


Ans. (4)

$KE = \frac{1}{2}mA^2\omega^2(Q^2\omega t)$

13. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer ? (speed of light = 3×10^8 ms^{-1})

(1) 10.1 GHz (2) 12.1 GHz

(3) 17.3 GHz (4) 15.3 GHz

Ans. (3)

14. An electric dipole has a fixed dipole moment $\vec{p}$, which makes angle θ with respect to x-axis. When subjected to an electric field $\overrightarrow{E_1} = E\hat{i}$, it experiences a torque $\overrightarrow{T_1} = \tau\hat{k}$. When subjected to another electric field $\overrightarrow{E_2} = \sqrt{3}E_1\hat{j}$ it experiences a torque $\overrightarrow{T_2} = -\overrightarrow{T_1}$. The angle θ is :

(1) 30° (2) 45°

(3) 60° (4) 90°

Ans. (3)

$\hat{\tau} = \hat{p} \times \hat{E}$

$\Rightarrow \theta = 60°$

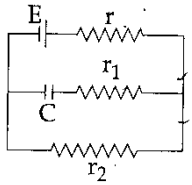
15. A capacitance of 2 μF is required in an electrical circuit across a potential difference of 1.0 kV. A large number of 1 μF capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is :

(1) 2 (2) 16

(3) 24 (4) 32

Ans. (4)

16. In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitance C will be :



(1) CE (2) $CE\frac{r_1}{(r_2 + r)}$

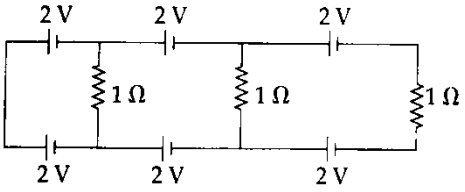
(3) $CE\frac{r_2}{(r + r_2)}$ (4) $CE\frac{r_1}{(r_1 + r)}$

Ans. (3)

$$V_c = \frac{Er_2}{r + r_2}$$

$$q_c = \frac{cEr_2}{r + r_2}$$

17.

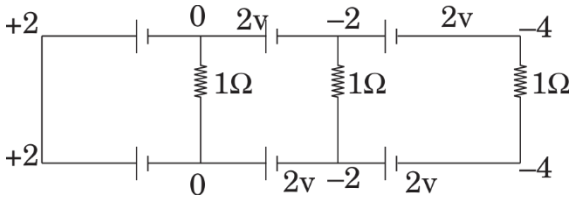


In the above circuit the current in each resistance is :

(1) 1 A (2) 0.25 A

(3) 0.5 A (4) 0 A

Ans. (4)



18. A magnetic needle of magnetic moment 6.7×10^{-2} Am^2 and moment of inertia 7.5×10^{-6} kg m^2 is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is :

(1) 6.65 s (2) 8.89 s

(3) 6.98 s (4) 8.76 s

Ans. (1)

$$T = 2\pi\sqrt{\frac{I}{\mu B}} = 0.6647 \text{ sec}$$

$t = 10T = 6.65$ sec.

19. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15 Ω, it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into a voltmeter of range 0 – 10 V is :

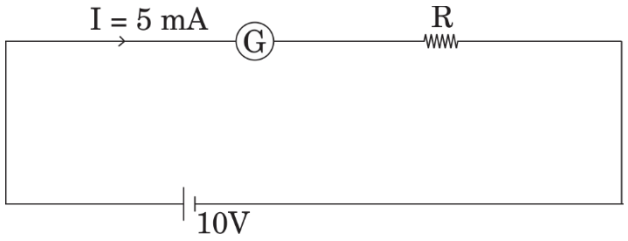
(1) 1.985×10^3 Ω

(2) 2.045×10^3 Ω

(3) 2.535×10^3 Ω

(4) 4.005×10^3 Ω

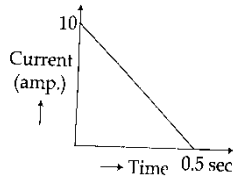
Ans. (1)



$5 \times 10^{-3} (15 + R) = 10$

$R = 1.985 \times 10^{+3}$ Ω

20. In a coil of resistance 100 W, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is :



(1) 200 Wb (2) 225 Wb

(3) 250 Wb (4) 275 Wb

Ans. (3)

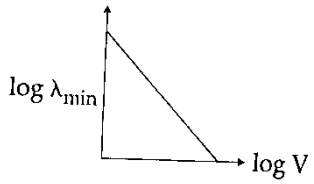
$\Delta\phi = R(\Delta q)$

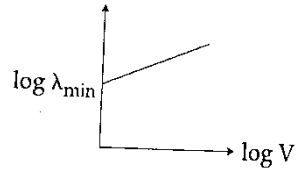
= 250 Wb.

21. An electron beam is acceleration by a potential difference V to hit a metallic target to produce X-rays. It produce continuous as well as characteristic X-rays. If λ_{min} is the smallest possible wavelength of X-ray in the spectrum, the variation of log λ_{min} with log V is correctly represented in :

(1)

(2)





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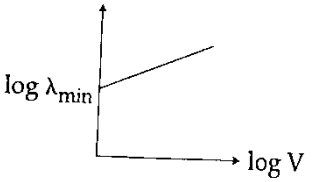

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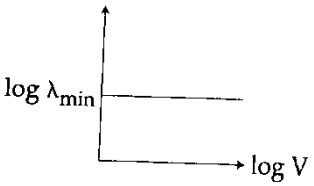


Ans. (1)

$$ev = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{ev}$$

$$\log\lambda = \log\left(\frac{hc}{e}\right) - \log v$$

22. A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed is

(1) real and at a distance of 40 cm from convergent lens

(2) virtual and at a distance of 40 cm from convergent lens

(3) real and at a distance of 40 cm from the divergent lens

(4) real and at a distance of 6 cm from the convergent lens

Ans. (1)

Use less formulae

$$\frac{1}{v} - \frac{1}{u} = 1/f$$

23. In a Young's double slit experiment, slits are separated by 0.5 mm, and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to the common central maximum to the point where the bright fringes due to both the wavelength coincide is :

(1) 1.56 mm (2) 7.8 mm

(3) 9.75 mm (4) 15.6 mm

Ans. (2)

$y = \frac{n\lambda D}{d}$ for bright frings

$n_1\lambda_1 = n_2\lambda_2$

$\frac{n_1}{n_2} = 4/5$

$\Rightarrow n_1 = 4, n_2 = 5$

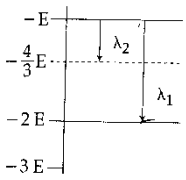
24. A particle A of mass m and initial velocity v collides with a particle B of mass $\frac{m}{2}$ which is at rest. The collision is head on, and elastic. The ratio of the de-Broglie wavelength λ_A to λ_B after the collision is :

(1) $\frac{\lambda_A}{\lambda_B} = \frac{1}{3}$ (2) $\frac{\lambda_A}{\lambda_B} = 2$

(3) $\frac{\lambda_A}{\lambda_B} = \frac{2}{3}$ (4) $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$

Ans. (2)

25. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths $r = \lambda_1 / \lambda_2$, is given by :



(1) $r = \frac{4}{3}$ (2) $r = \frac{2}{3}$

(3) $r = \frac{3}{4}$ (4) $r = \frac{1}{3}$

Ans. (4)

$\lambda_1 = \frac{hc}{\Delta E} = \frac{hc}{E}$

$\lambda_2 = \frac{3hc}{E} \Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{1}{3}$

26. A radioactive nucleus A with a half life T, decays into a nucleus B. At t = 0, there is no nucleus B. At sometime t, the ratio of the number of B to that of A is 0.3. Then, t is given by :

(1) $t = \frac{T}{2}\frac{\log 2}{\log 1.3}$ (2) $t = T\frac{\log 2}{\log 2}$

(3) $t = T \log (1.3)$ (4) $t = \frac{T}{\log(1.3)}$

Ans. (2)

$N = N_0 e^{-\lambda t}$

$N_0 = 1.3\ N$

$\frac{N_0}{N} = 1.3 = e^{\lambda t}$

$\lambda t = \log(1.3)$

$\frac{\log 2}{T} t = \log(1.3)$

$t = \frac{T \log(1.3)}{\log 2}$

27. In a common emitter amplifier circuit using an n-p-n transistor, the phase difference between the input and the output voltage will be :

(1) 45° (2) 90°

(3) 135° (4) 180°

Ans. (4)

28. In amplitude modulation, sinusoidal carrier frequency used is denoted by ω_c and the signal frequency is denoted by ω_m. The bandwidth ($\Delta\omega_m$) of the signal is such the $\Delta\omega_m << \omega_c$. Which of the following frequencies is not contained in the modulated wave?

(1) ω_m (2) ω_c

(3) $\omega_m + \omega_c$ (4) $\omega_c - \omega_c$

Ans. (1)

Formula based.

29. Which of the following statements is false?

(1) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude

(2) In a balanced wheatstone bridge if the cell and the galvanometer are exchanged, the null point is disturbed

(3) A rheostat can be used as a potential divider

(4) Kirchhoff's second law represents energy conservation

Ans. (2)

30. The following observations were taken for determining surface tension T of water by capillary method : diameter of capillary, D = 1.25×10^{-2} m rise of water, h = 1.45×10^{-2} m. Using g = 9.80 m/s^2 and the simplified relation T = $\frac{rhg}{2} \times 10^3$ N/m, the possible error in surface tension is closest to :

(1) 0.15% (2) 1.5%

(3) 2.4% (4) 10%

Ans. (2)

$$\frac{\Delta T}{T} = \frac{\Delta r}{r} + \frac{\Delta h}{h} + \frac{\Delta g}{g} = 1.5\%$$

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PART B - CHEMISTRY

31. Given

$C(graphite) + O_2(g) \rightarrow CO_2(g)$:

$\Delta rH° = 393.5\ kJ\ mol^{-1}$

$H_2(g) + \frac{1}{2}O_2(g)\ H_2O(l)$:

$\Delta rH° = -285.8\ kJ\ mol^{-1}$

$CO_2(g) + 2H_2O(l) \rightarrow CH_4(g) + 2O_2(g)$:

$\Delta_r H^o = +890.3\ kJ\ mol^{-1}$

Based on the above thermochemical equations, the value of $\Delta_r H°$ at 298 K for the reaction the reaction

$C(graphite)\ 2H_2(g) - CH_4(g)$ will be:

(1) $-74.8\ kJ\ mol^{-1}$

(2) $-144.0\ kJ\ mol^{-1}$

(3) $+74.8\ kJ\ mol^{-1}$

(4) $+144.8\ kJ\ mol^{-1}$

Ans. (1)

$890.3 = 2\Delta H_f(O_2) + 1 \times \Delta H_f(CH_4) - \Delta H_f(O_2) - 2 \times \Delta H_f(H_2O)$

$890.3 = 0 + 2 - (-393.5) - 2(-285.8)$

$x = 890.3 - 393.5 - 571.6$

$= -74.8\ kJ/mol$

32. 1 gram of a carbonate (M_2CO_3) on treatment with excess HCl produces 0.01186 mole of CO_2. The molar mass of M_2CO_3 in $g\ mol^{-1}$ is:

(1) 118.6

(2) 11.86

(3) 1186

(4) 84.3

Ans. (4)

$$\underset{1\,gm}{M_2CO_3} \rightarrow \underset{0.01186 mol}{CO_2}$$

$n_{M_2CO_3} = n_{CO_2}$

$\frac{1}{M} = 0.01186$

$M = \frac{1}{0.01186} = 84.3$

33. ΔU is equal to:

(1) Adiabatic work

(2) Isothermal work

(3) Isochoric work

(4) Isobaric work

Ans. (1)

$\Delta U = q + w$

Adiabatic process $q = 0$

$\Delta U = w$

34. The Tyndall effect is observed only when following conditions are satisfied :

(a) The diameter of the dispersed particle is much smaller than the wavelength of the light used.

(b) The diameter of the dispersed particle is not much smaller than the wavelength of the light used.

(c) The refractive indices of the dispersed phase and dispersion medium are almost similar in magnitude.

(d) The refractive indices of the dispersed phase and dispersion medium differ greatly in magnitude.

(1) (a) and (c)

(2) (b) and (c)

(3) (a) and (d)

(4) (b) and (d)

Ans. (4)

Refer NCERT page no. 139 (Chemistry Book No. 1)

35. A metal crystallises in a face centred cubic structure. If the edge length of its unit cell is 'a', the closest approach between two atoms in metallic crystal will be :

(1) $\sqrt{2}$ a (2) $\frac{a}{\sqrt{2}}$

(3) 2a (4) $2\sqrt{2}$ a

Ans. (2)

Distance of closest approach is half of face diagonal = $\frac{\sqrt{2}a}{2} = \frac{a}{\sqrt{2}}$

36. Given

$E^{\circ}_{Cl_2/Cl^-} = 1.36V, E^{\circ}_{Cr^{3+}/Cr} = -0.74V$

$E^{\circ}_{Cr_2O_7^{2-}/Cr^{3+}} = 1.33V, E^{\circ}_{MnO_4^-/Mn^{2+}} = 1.51\ V$

Among the following, the strongest reducing agent is:

(1) Cr^{3+} (2) Cl^-

(3) Cr (4) Mn^{2+}

Ans. (3)

Higher is –ve value of standard reduction potential better is the reducing agent

37. The freezing point of benzene decreases by 0.45° C when 0.2 g of acetic acid is added to 20 g of benzene. If acetic acid associates to form a dimer in benzene, percentage association of acetic acid in benzene will be:.

(K_f for benzene = 5.12 K kg mol^{-1})

(1) 74.6% (2) 94.6%

(3) 64.6% (4) 80.4%

Ans. (2)

$\Delta T_f = i\ K_f\ m$

$0.45 = i \times 5.12 \times \frac{\frac{0.2}{60}}{20} \times 1000$

$i = 0.527 = 1 + \left(\frac{1}{n} - 1\right)\beta$

$0.527 = 1 + \left(\frac{1}{n} - 1\right)\beta$

$\beta = 0.946$ or 94.6%

38. The radius of the second Bohr orbit for hydrogen atom is:

(Planck's Const. h = 6.6262×10^{-34}Js; mass of electron = 9.1091×10^{-31} kg; charge of electron e = 1.60210×10^{-19} C; permittivity of vacuum

$\in_0 = 8.854185 \times 10^{-12}\ kg^{-1}\ m^{-3}\ A^2$)

(1) 0.529 Å (2) 2.12 Å

(3) 1.65 Å (4) 4.76 Å

Ans. (2)

$r = r_0 \times \frac{n^2}{7} = 0.529 \times \frac{2^2}{1} = 2.12$ Å

39. Two reaction R_1 and R_2 have identical pre-exponential factors. Activation energy of R_1 exceeds that of R_2 by 10 kJ mol^{-1}. If k_1 and k_2 are rate constants for reactions R_1 and R_2 respectively at 300 K, then In (k_2/k_1) is equal to :

(1) 6 (2) 4

(3) 8 (4) 12

Ans. (2)

$k_1 = A\ e^{-E_1/RT}$

$k_2 = A\ e^{-E_2/RT}$

$\frac{k_1}{k_2} = e^{-(E_1-E_2)/RT}$

$\frac{k_2}{k_1} = e^{(E_1-E_2)/RT}$

$\text{In}\ \frac{k_2}{k_1} = \frac{E_1 - E_2}{RT} = \frac{10\times1000}{8.314\times300}$

$= 4.009$

40. pK_a of a weak aicd (HA) and pK_b of a weak bas (BOH) are 3.2 and 3.4, respectively. The pH of their salt (AB) solution is:

(1) 7.0 (2) 1.0

(3) 7.2 (4) 6.9

Ans. (4)

$pH = 7 + \frac{1}{2}(pK_a - pK_b)$

$= 7 + (3.2 - 3.4) = 7 + \frac{-0.2}{2}$

$= 7 - 0.1 = 6.9$

41. Both lithium and magnesium disply several similar properties due to the diagonal relationship; however, the one which is incorrect is :

(1) both form nitrides

(2) nitrates of both Li and Mg yield of NO_2 and O_2 on heating

(3) both form basic carbonates

(4) both form soluble bicarbonates

Ans. (3)

Lithium doesn't form basic carbonates

42. Which of the following species is not paramagnetic?

(1) O_2 (2) B_2

(3) NO (4) CO

Ans. (4)

CO has $14e^-$ and all are paired and is hence diamagnetic

43. Which of the following reactions is an example of a redox reaction?

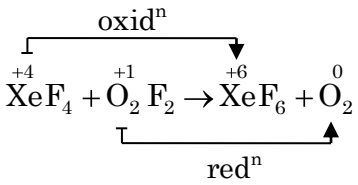
(1) $XeF_6 + H_2O \rightarrow XeOF_4 + 2HF$

(2) $XeF_6 + 2H_2O \rightarrow XeO_2F_2 + 4HF$

(3) $XeF_4 + O_2F_2 \rightarrow XeF_6 + O_2$

(4) $XeF_2 + PF_5 \rightarrow [XeF]^+ PF_6^-$

Ans. (3)



44. A water sample has ppm level concentration of following anions

$F^- = 10;\ SO_4^{2-} = 100;\ NO_3^- = 50$

The anion/anions that make/makes the water sample unsuitable for drinking is / are :

(1) only F^-

(2) only SO_4^{2-}

(3) only NO_3^-

(4) both SO_4^{2-} and NO_3^-

Ans. (1)

Permissible limits (as per NCERT)

$SO_4^{2-} = 500$ ppm

$NO_3^- = 50$ ppm

$F^- = 2$ppm

45. The group having isoelectronic species is:

(1) O^{2-}, F^-, Na, Mg^{2+}

(2) O^-, F^-, Na^+, Mg^{2+}

(3) O^{2-}, F^-, Na^+, Mg^{2+}

(4) O^-, F^-, Na, Mg^+

Ans. (3)

O^{2-}, F^-, Na^+, Mg^{2+} contain $10e^-$ each.

46. The products obtained when chlorine gas reacts with cold and dilute aqueous NaOH are:

(1) Cl^- and ClO^-

(2) Cl^- and ClO_2^-

(3) ClO^- and ClO_3^-

(4) ClO_2^- and ClO_3^-

Ans. (1)

Cl_2 + cold & dil. NaOH $\rightarrow$ NaCl + NaClO

Cl^- & ClO^- are formed

47. In the following reactions, ZnO is respectively acting as a / an:

(a) $ZnO + Na_2O \rightarrow Na_2ZnO_2$

(b) $ZnO + CO_2 \rightarrow ZnCO_3$

(1) acid and acid

(2) acid and base

(3) base and acid

(4) base and base

Ans. (2)

$$\underset{\text{acidic oxide}}{ZnO} + Na_2O \rightarrow Na_2ZnO_2$$

$$\underset{\text{basic oxide}}{ZnO} + CO_2 \rightarrow ZnCO_3$$

48. Sodium salt of an organic acid 'X' produces effervescence with conc. H_2SO_4. 'X' reacts with the acidified aqueous $CaCl_2$ solution to give a white precipitate which decolourises acidic solution of $KMnO_4$. 'X' is :

(1) CH_3COONa

(2) $Na_2C_2O_4$

(3) C_6H_5COONa

(4) HCOONa

Ans. (4)

$$\underset{(X)}{Na_2C_2O_4} + \text{conc. } H_2SO_4 \rightarrow Na_2SO_4 + H_2O + CO + \underset{\text{effervescence}}{CO_2\uparrow}$$

$$\underset{(X)}{Na_2C_2O_4} + CaCl_2 \rightarrow \underset{\text{white ppt}}{CaC_2O_4\downarrow} + 2NaCl$$

$$H^+ + C_2O_4^- + MnO_4^- \rightarrow CO_2 + Mn^{2+} + H_2O$$

49. The most abundant elements by mass in the body of a healthy human adult are:

Oxygen (61.4%); Carbon (22.9%), Hydrogen (10.0%); and Nitrogen (2.6%). The weight which a 75 kg person would gain if all ^1H atoms are replaced by ^2H atoms is :

(1) 7.5 kg (2) 10 kg

(3) 15 kg (4) 37.5 kg

Ans. (1)

Weight of H_2 already present

= 10% of 75 kg = 7.5 kg

As ^1H is replaced by ^2H weight of H_2 is doubled.

$\therefore$ gain in weight = 7.5

50. On treatment of 100 mL of 0.1 M solution $CoCl_3.6H_2O$ with excess $AgNO_3$; 1.2×10^{22} ions are precipitated. The complex is :

(1) $[Co(H_2O)_6]Cl_3$

(2) $[Co(H_2O)_5Cl]Cl_2.H_2O$

(3) $[Co(H_2O)_4Cl_2]Cl.2H_2O$

(4) $[Co(H_2O)_3Cl_3].3H_2O$

Ans. (2)

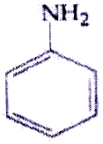
$$n_{complex} = \frac{100}{1000} \times 0.1 = 0.01$$

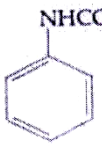
$$n_{ppt} = \frac{1.2 \times 10^{22}}{6 \times 10^{23}} = 0.02$$

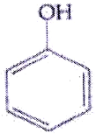
$\therefore$ each mole of complex has 2 moles of ionisable Cl^-.

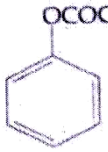
Hence formula of complex is $[Co(H_2O)_5Cl]Cl_2 \cdot H_2O$

51. Which of the following compound will form significant amount of *meta* product during mono-nitration reaction?

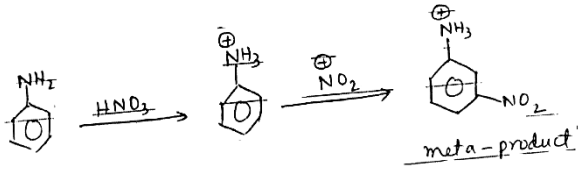
(1) 

(2) 

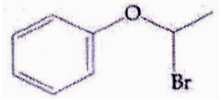
(3) 

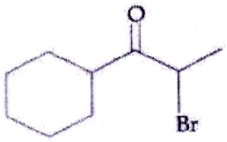
(4) 

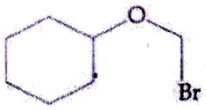
Ans. (1)

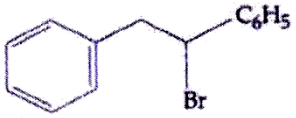


52. Which of the following, upon treatment which *tert*-BuONa followed by addition of bromine water, fails to decolourize the colour of bromine?

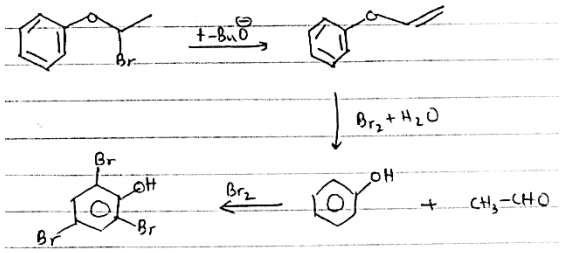
(1) 

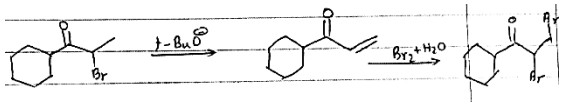
(2) 

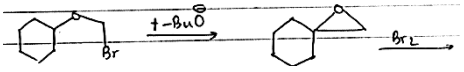
(3) 

(4) 

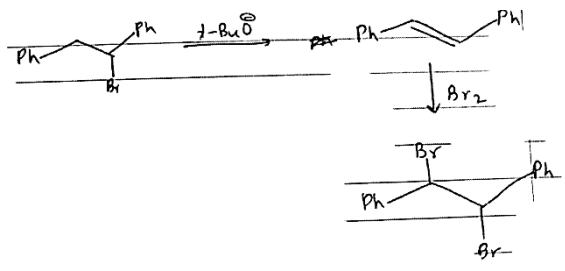
Ans. (3)

(i) 

(2) 

(3) 

no reaction

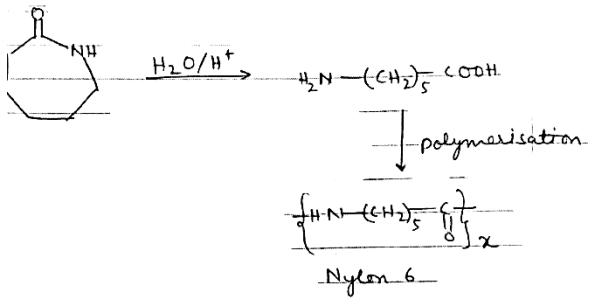
(4) 

53. The formation of which of the following polymers involves hydrolysis reaction?

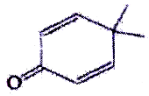
(1) Nylon 6, 6 (2) Terylene

(3) Nylon 6 (4) Bakelite

Ans. (3)

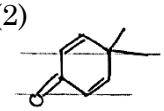


54. Which of the following molecules is least resonance stabilized?

(1)  (2) 

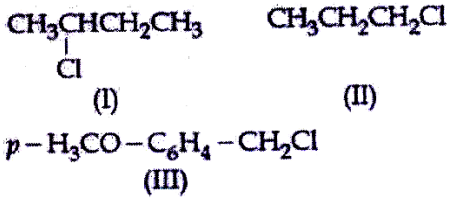
(3)  (4) 

Ans. (2)



Non aromatic, hence least stabilized by resonance.

55. The increasing order of the reactivity of the following halides for the S_N1 reaction is:



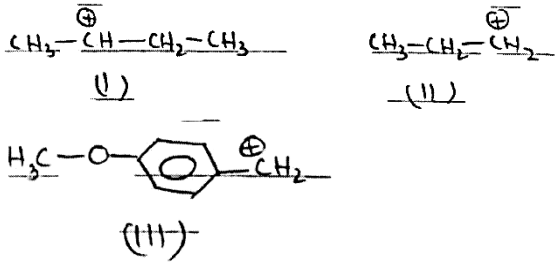
(1) (I) < (III) <(II)

(2) (II) < (III) < (I)

(3) (III) < (II) < (I)

(4) (II) < (I) < (III)

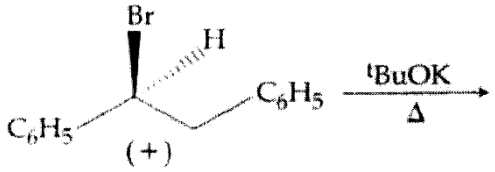
Ans. (4)



Stability of carbocation: III > I > II

Rate of S_N1 : III > I > II

56. The major product obtained in the following reaction is:



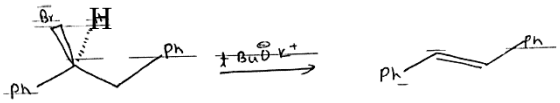
(1) (+)$C_6H_5CH(O^tBu)CH_2C_6H_5$

(2) (–)$C_6H_5CH(O^tBu)CH_2C_6H_5$

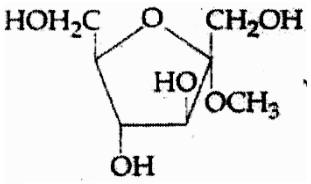
(3) (±)$C_6H_5CH(O^tBu)CH_2C_6H_5$

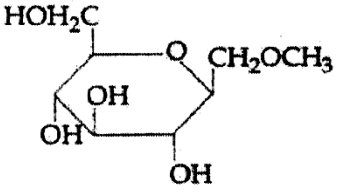
(4) $C_6H_5CH = CHC_6H_5$

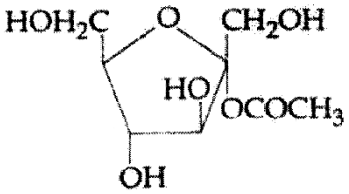
Ans. (4)

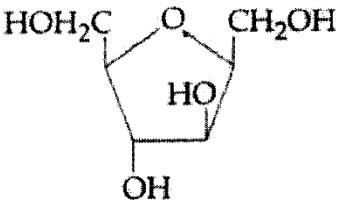


57. Which of the following compounds will behave as a reducing sugar in an aqueous KOH solution?

(1) 

(2) 

(3) 

(4) 

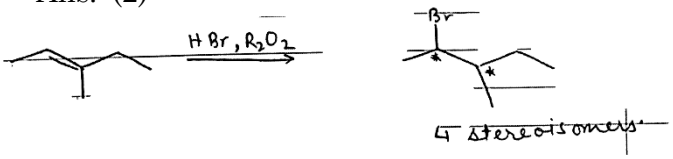
Ans. (3)

58. 3-Methyl-pent-2-ene on reaction with HBr in presence of peroxide forms an addition product. The number of possible stereoisomers for the product is :

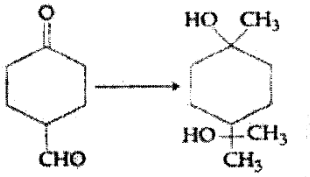
(1) Two (2) Four

(3) Six (4) Zero

Ans. (2)



59. The correct sequence of reagents for the following conversion will be:



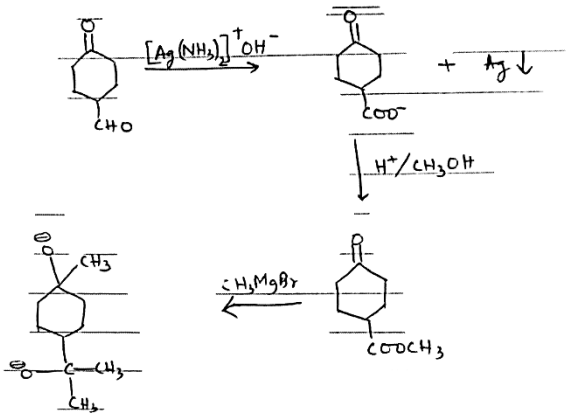
(1) CH_3MgBr, $[Ag(NH_3)_2]^+ OH^-$, H^+/CH_3OH.

(2) $[Ag(NH_3)_2]^+ OH^-$, H^+/CH_3OH, H^+ / CH_3OH

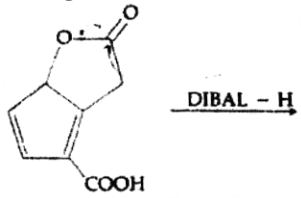
(3) $[Ag(NH_3)_2]^+OH^-$, H^+/CH_3OH, CH_3MgBr

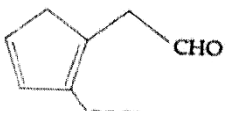
(4) CH_3MgBr, H^+/CH_3OH, $[Ag(NH_3)_2]^+ OH^-$

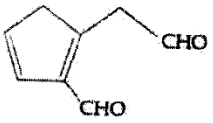
Ans. (3)

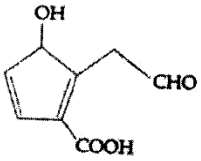


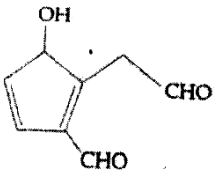
60. The major product obtained in the following reaction is :



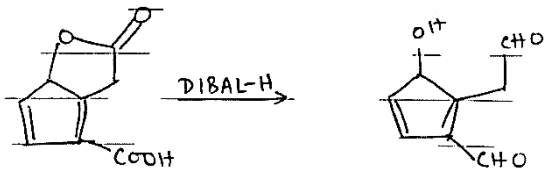
(1) 

(2) 

(3) 

(4) 

Ans. (4)



Part C – Mathematics

61. The function $f : R - \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x) = \frac{x}{1+x^2}$, is

(1) injective but not surjective.

(2) surjective but not injective

(3) neither injective nor surjective (4) invertible.

Ans. (4)

$f : R\left[\frac{1}{2}, \frac{1}{2}\right]$

$f(x) = \frac{x}{1+x^2}$

$f^{-1}(x) = \frac{(1+x^2) - x(2x)}{1+x^2}$

$= \frac{1-x^2}{(1+x^2)^2} > 0$ for $x \in (-1,1)$

< 0 for $x \in (-\infty,-1) \cup (-1,1)$

Not one-one

$\frac{x}{1+x^2} = y \quad x = y + x^2y$

$x^2y - x + y = 0$

$1 - 4y^2 \geq 0 \quad y \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

62. If for a positive integer n, the quadratic equation,

$x(x + 1) + (x + 1)(x + 2) + \ldots\ldots\ldots$

$+ (x + \overline{n-1})(x + n) = 10n$ has two consecutive integral solution then n is equal to :

(1) 9 (2) 10

(3) 11 (4) 12

Ans. (1)

$\Sigma x^2 + (2n-1)n + (n^2 - n) = 10n$

$nn^2 + n^2n + \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} = 10n$

$nn^2 + n^2n + \frac{n(n+1)}{6}[2n + 1 - 3] = 10n$

$nn^2 + n^2n + \frac{n(n+1)}{6}2[n-1] = 10n$

$nn^2 + n^2n + \frac{n(n^2-1)}{6}2[n-1] = 10n$

$nn^2 + n^2n + \frac{n^3 - n - 30n}{3} = 0$

$n^4 - 4n\frac{(n^3 - n - 30n)}{3} = n^2$

$3n^4 - 4n^4 + 124n^2 = 3n^2$

$n^4 - 121n^2 = 0 \quad [n^2[n^2 - 121] = 0 \quad n = 11$

63. let ω be a complex number such that $2\omega + 1 = z$ where $z = \sqrt{-3}$. If

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k,$$

Then k is equal to :

(1) z (2) –1

(3) 1 (4) –z

Ans. (2)

$2\omega + 1 = z$

$z = \sqrt{-3} = \sqrt{(3)(-1)} = \sqrt{3}i$

$2\omega + 1 = 3i \quad \omega = \frac{-1 + \sqrt{3}i}{2}$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix} \quad \begin{vmatrix} 3 & 1 & 1 \\ 0 & \omega & \omega^2 \\ 0 & \omega^2 & \omega \end{vmatrix}$$

$= 3(\omega^2 - \omega^4) = 3(\omega^2 - \omega)$

$= 3(-i\sqrt{3}) = 3(\omega^2 - \omega)$

$k = -z$

64. If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then adj $(3A^2 + 12A)$ is equal to :

(1) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$ (2) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$

(3) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$ (4) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$

Ans. (3)

$A^2 = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$

$= \begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix}$ $3A^2 + 12A$

$= \begin{bmatrix} 48 & -27 \\ -36 & 39 \end{bmatrix} + \begin{bmatrix} 24 & -36 \\ -48 & 12 \end{bmatrix}$

$= \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

$\text{ADJ}(3A^2 + 12A) = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

65. If S is the set of distinct values of 'b' for which the following system of linear equations

$x + y + z = 1$

$x + ay + z = 1$

$ax + by + z = 0$

has no solution, then S is :

(1) an infinite set

(2) a finite set containing two or more elements

(3) a singleton

(4) an empty set

Ans. (1)

$\begin{bmatrix} 1 & 1 & 1 & 1 \\ a & b & 1 & 0 \\ 1 & a & 1 & 1 \end{bmatrix} R_2 \to R_2 - R_1 \;\; R_3 \to R_3 - R_1$

$\begin{bmatrix} 1 & 1 & 1 & 1 \\ a-1 & b-1 & 0 & -1 \\ 0 & a-1 & 0 & 0 \end{bmatrix}$

If $a = 1$ $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & b-1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ if $a \neq 1$

If $b = 1$ no solution.

66. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party is :

(1) 468 (2) 469

(3) 484 (4) 4854

Ans. (2)

$\frac{4(X)}{4L,3M}$		$\frac{W(Y)}{3L,4M}$		3L, 3M
0	3	3	0	1*1*1*1=1
1	2	2	1	4*3*3*4=144
2	1	1	2	6*3*3*6=324
3	0	0	3	4*1*1*4=16
				485

67. The value of

$(^{21}C_1 - {}^{10}C_1) + (^{21}C_2 - {}^{10}C_2) +$

$(^{21}C_3 - {}^{10}C_3) + (^{21}C_4 - {}^{10}C_4) + \ldots. +$

$(^{21}C_{10} - {}^{10}C_{10})$ is :

(1) $2^{21} - 2^{10}$ (2) $2^{20} - 2^9$

(3) $2^{20} - 2^{10}$ (4) $2^{21} - 2^{11}$

Ans. (1)

$(^{21}C_1 + {}^{21}C_2 + \ldots + {}^{21}C_{10}) - (^{10}C_1 + {}^{10}C_2 \ldots + {}^{10}C_{10})$

$= (2^{20} - 1) - (2^{10} - 1)$

$= 20^{20} - 2^{10}$

68. For any three positive real numbers a, b and c.

$9(25a^2 + b^2) + 25(c^2 - 3ac)$

$= 15b\,(3a + c)$.

(1) b, c and a are in A.P

(2) a, b and c are in A.P

(3) a, b and c are in G.P

(4) b, c and a are in G.P

Ans. (3)

$$\frac{25a^2 + b^2}{25} + \frac{c^2 - 3ac}{9} = \frac{b(3a + c)}{15}$$

$$a^2 + \frac{b^2}{25} + \frac{c^2}{9} = \frac{ac}{3} + \frac{ab}{5} + \frac{bc}{15}$$

$$a^5 + \left(\frac{b}{5}\right)^2 + \left(\frac{c}{3}\right)^2 = a.\frac{c}{3} + a.\frac{b}{5} + \frac{b}{5} + \frac{b}{5}.\frac{c}{3}$$

$$\frac{a}{1} = \frac{b}{5} = \frac{c}{3} = \lambda \qquad \lambda, 3\lambda, 5\lambda$$

a, c, b or b c a are in A.P.

69. Let a, b, c $\in$ R. If $f(x) = ax^2 + bx + c$ is such that $a + b + c = 3$ and

$f(x + y) = f(x) + f(y) + xy, \ \forall \ x, y \in R$,

then $\sum_{n=1}^{10} f(n)$ is equal to :

(1) 165 (2) 190

(3) 255 (4) 330

Ans. (2)

$f(x + y) = a(x + y)^2 + b(x + y) + c$

$= (ax^2 + bx + c) + (ay^2 + by + c) + (2axy - c)$

$= f(x) + f(y) + xy \Rightarrow c = 0$ & $a = \frac{1}{2}$

$$\because a + b + c = 3 \Rightarrow b = 3 - \frac{1}{2} = \frac{5}{2}$$

$$f(x) = \frac{1}{2}x^2 + \frac{5x}{2}$$

$$\sum_{n=1}^{10} f(x) = \frac{1}{2}\sum_{n=1}^{10} n^2 + \frac{5}{2}\sum_{n=1}^{10} n$$

$$= \frac{1}{2}\frac{10(11)(21)}{6} + \frac{5}{2} \times \frac{10 \times 11}{2}$$

$$= \frac{385}{2} + \frac{275}{2} = \frac{660}{} = 330$$

70. $\lim\limits_{x \to \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - (2x)^3}$ equals :

(1) $\frac{1}{16}$ (2) $\frac{1}{8}$

(3) $\frac{1}{4}$ (4) $\frac{1}{24}$

Ans. (3)

$$\lim_{x \to \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$$

$$\lim \frac{-\tan t + \sin t}{(-2t)^3}$$

$$= \lim_{t \to 0}\left(\frac{\tan t - \sin t}{8t^3}\right)$$

$$= \frac{1}{8} \times \frac{1}{2} = \frac{1}{16}$$

71. If for $x \in \left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1 - 9x^3}\right)$ is $\sqrt{x} \cdot g(x)$, then g (x) equals :

(1) $\frac{3x\sqrt{x}}{1 - 9x^3}$ (2) $\frac{3x}{1 - 9x^3}$

(3) $\frac{3}{1 + 9x^3}$ (4) $\frac{9}{1 + 9x^3}$

Ans. (2)

$$\tan^{-1}\left(\frac{6x\sqrt{x}}{1 - 9x^3}\right) = 2\tan^{-1} 3x\sqrt{x}$$

$$f^{-1}(x) = \frac{9\sqrt{x}}{1 + 9x^3}$$

$$g(x) = \frac{9}{1+9x^3}$$

72. The normal to the curve $y(x-2)(x-3) = x+6$ at the point where the curve intersects the y-axis passes through the point :

(1) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (2) $\left(\frac{1}{2}, -\frac{1}{3}\right)$

(3) $\left(\frac{1}{2}, \frac{1}{3}\right)$ (4) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

Ans. (3)

$y(x-2)(x-3) = x+6,\ y = \frac{6}{6} = 1$

$$y = \frac{x+6}{x^2-5x+6}$$

$$y^1 = \frac{(x^2-5x+6)-(x+6)(2x-5)}{(x^2-5x+6)^2}$$

$$= \frac{(x^2-5x+6)-(2x^2+7x-30)}{(x^2-5x+6)^2}$$

$$y^1 = \left|\frac{-x^2-12x+36}{(x^2-5x+6)^2} = (x^2-5x+6)\right|_{x=0} = 1$$

$y - 1 = -1(x-0) \Rightarrow y = -x + 1$

$$\frac{-(2x^2+7x-30)}{(x^2-5x+6)}$$

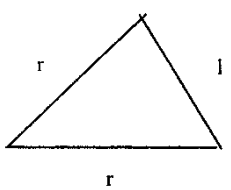
$\therefore$ normal slope = –1

73. Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower-bed, is :

(1) 10 (2) 25

(3) 30 (4) 12.5

Ans. (4)



$2r + r\theta = 20$

$r(2+\theta) = 20$

$$r = \frac{20}{2+\theta}\ \theta r = \frac{20-r}{r}$$

$$\frac{dA}{dr} = 10 - 2r = 0\ 2r = 10\ r = s$$

$$A = \frac{1}{2}r^2\left(\frac{20-2r}{r}\right) = \frac{1}{2}r(20-2r)$$

$$= \frac{1}{2}(20r - 2r^2) = 10r - r^2$$

$$A = \frac{1}{2}r^2\theta \quad \theta = \frac{20-10}{5} = \frac{10}{5} = 2$$

$$= \frac{1}{2}25 \times 2$$

$A = 25$

74. Let $I_n = \int \tan^n x\, dx$, $(n > 1)$.

$I_n + I_6 = a\tan^5 x + bx^5 + C$, where C, is a constant of integration, then the ordered pair (a, b)is equal to :

(1) $\left(\frac{1}{5}, 0\right)$ (2) $\left(\frac{1}{5}, -1\right)$

(3) $\left(-\frac{1}{5}, 0\right)$ (4) $\left(-\frac{1}{5}, 1\right)$

Ans. (3)

$$I_n = \int \mathrm{Tan}^n x$$

$$I_4 + I_6 = \int \mathrm{Tan}^4 x + \int \mathrm{Tan}^6 x$$

$$= \int t^4 dt = \frac{t^5}{5} + c = \frac{1}{5}\tan^5 x + c$$

$= a\tan^5 x + bx^5 + c$

$$\Rightarrow a = \frac{1}{5}, b = 0$$

75. The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to :

(1) 2 (2) 4

(3) –1 (4) –2

Ans. (3)

$$\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x} \quad I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1}{1-\cos x}$$

$$2I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \left(\frac{1}{1+\cos x} + \frac{1}{1-\cos x} \right)$$

$$= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \left(\frac{1-\cos x+1+\cos x}{\sin^2 x} \right)$$

$$2I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} 2\,\text{cosec}^2 x$$

$$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (-\cot x) = -\left(\cot\frac{3\pi}{4} - \cot\frac{\pi}{4} \right)$$

$= -(-1 - 1) = 2$

76. The area (in sq. units) of the region $\{(x, y) : x \geq 0, x + y \leq 3, x^2 \leq 4y$ and $y \leq 1 + \sqrt{x}\}$ is :

(1) $\frac{3}{2}$ (2) $\frac{7}{3}$

(3) $\frac{5}{2}$ (4) $\frac{59}{12}$

Ans. (1)

$$\text{Area} = \int_0^1 (1+\sqrt{x})dx + \int_1^2 (3-x)dx - \int_0^2 \frac{x^2}{4}dx$$

$$= \left[x + \frac{2}{3}x^{\frac{3}{2}} \right]_0^1 + \left[3x - \frac{x^2}{2} \right]_1^2 - \frac{1}{12}[x^3]_0^2$$

$$= 1 + \frac{2}{3} + 4 - 3 + \frac{1}{2} - \frac{2}{3}$$

$$= \frac{5}{2}$$

77. If $(2 + \sin x)\frac{dy}{dx} + (y + 1)\cos x = 0$ and $y(0) = 1$, then $y\left(\frac{\pi}{2}\right)$ is equal to :

(1) $-\frac{2}{3}$ (2) $-\frac{1}{3}$

(3) $\frac{1}{3}$ (4) $\frac{1}{3}$

Ans. (2)

$$(2 + \sin x)\frac{dy}{dx} + (y + 1)\cos x = 0$$

$$\frac{2+\sin x}{\cos x} + (y+1)\frac{dy}{dx} = 0$$

$l\text{n}(y + 1) + l\text{n}(2 + t) = l\text{n}\, c$

$(y + 1) + (2 + \sin x) = c \quad x = 0$

$(1 + 1)(2) = C \Rightarrow C = 4$

$(y + 1) + (2 + \sin x) = 4$

$$x = \frac{\pi}{2} \Rightarrow (y + 1) + (2 + 1) = 4$$

$$y + 1 = \frac{4}{3} \Rightarrow y = \frac{4}{3} - 1 = \frac{1}{3}$$

78. Let k be an integer such that the triangle with vertices (k, –3k), (5, k) and (–k, 2) has area 28 sq. units. Then the orthocenter of this triangle is at the point :

(1) $\left(1, \frac{3}{4}\right)$ (2) $\left(1, -\frac{3}{4}\right)$

(3) $\left(2, \frac{1}{2}\right)$ (4) $\left(2, -\frac{1}{2}\right)$

Ans. (1)

$$\frac{1}{2}\begin{vmatrix} k & 5 & -k & k \\ -3k & k & 2 & -3k \end{vmatrix} = 28$$

$$\frac{1}{2}|(k^2 + 15k) + 10 + k^2 + 3k^2 - 2k| = 28$$

$$\frac{1}{2}|5k^2 + 13k + 10| = 28$$

$$5k^2 + 13k + 10 = 56$$

$5k^2 + 13k - 46 = 0$

$5k^2 + 23k - 10k - 46 = 0$

$k(5k + 23) - 2(5k + 23) = 0$

$k = 2$ (or) $-\frac{23}{5}$

(2, –6) (5, 2) (–2, 2)

Substitution are

x = 2 and 3x + 8y –10 = 0

$\therefore\ H\left(2, \frac{1}{2}\right)$

79. The radius of a circle, having minimum area, which touches the curve $y = 5 - x^2$ and the lines, y = |x| is :

(1) $2(\sqrt{2} - 1)$ (2) $4(\sqrt{2} - 1)$

(3) $4(\sqrt{2} + 1)$ (4) $2(\sqrt{2} + 1)$

Ans. (4)

There are two circles satisfying the given conditions. The circle shown is of least area.

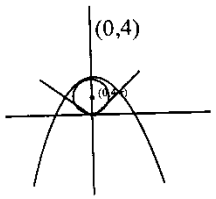
Let radius of circle is 'r'

$\therefore$ co-ordinates of centre = (0, 4 – r)

$\because$ circle touches the line y = x in first quadrant

$\therefore\ \left|\frac{0 - (4 - r)}{\sqrt{2}}\right| = r \Rightarrow r = 4 = \pm r\sqrt{2}$

$r = \frac{4}{\sqrt{2} + 1} = 4(\sqrt{2} - 1)r$



80. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$, If one of the directrices is x = –4, then the equation of the normal to it at $\left(1, \frac{3}{2}\right)$ is :

(1) 4x – 2y = 1 (2) 4x + 2y = 7

(3) x + 2y = 4 (4) 2y – x = 2

Ans. (3)

$C = \frac{1}{2}, ac = 4$

a = 2, b = $\sqrt{3}$

normal at $\left(1, \frac{3}{2}\right)$ is

$\frac{4x}{1} - \frac{3y}{3} \times 2 = 1$

4x – 2y – 1

81. A hyperbola passes through the point P($\sqrt{2}, \sqrt{3}$) and has foci at (±2, 0). Then the tangent to this hyperbola at P also passes through the point :

(1) $(3\sqrt{2}, 3\sqrt{3})$ (2) $(\sqrt{2}, \sqrt{3})$

(3) $(\sqrt{2}, -\sqrt{3})$ (4) $(\sqrt{2}, 2\sqrt{3})$

Ans. (3)

$P(\sqrt{2}, \sqrt{3})$ (±2, 0)

ae = 2 $b^2 = a^2(e^2 - 1) = 4 - a^2$

$\frac{x^2}{a^2} - \frac{y^2}{4 - a^2} = 1$

$\frac{2}{a^2} - \frac{3}{4 - a^2} = 1$

$8 - 2a^2 - 3a^2 = 4a^2 - a^4$

$a^4 - 9a^2 + 8 = 0$

$e^2 = \frac{4}{a^2} = 4, \frac{1}{2}$

$\because e > 1 \therefore e^2 = 4, a^2 = 1, b^2 = 3$

$x^2 - \frac{y^2}{3} = 1\,x$

Tgt. at $(\sqrt{2},\sqrt{3}) \equiv x\sqrt{2} - \frac{y}{\sqrt{3}} = 1$

$(2\sqrt{2},3\sqrt{3})$ statistics.

82. The distance of the point (1, 3, –7) from the plane passing through the point (1, –1, –1), having normal perpendicular to both the lines $\frac{x-1}{1} = \frac{y+2}{-2} = \frac{z-4}{3}$

and $\frac{x-2}{2} = \frac{y+1}{-1} = \frac{z+7}{-1}$, is :

(1) $\frac{10}{\sqrt{83}}$ (2) $\frac{5}{\sqrt{83}}$

(3) $\frac{10}{\sqrt{74}}$ (4) $\frac{20}{\sqrt{74}}$

Ans. (3)

$a(x - 1) + b(y + 1) + c(z + 1) = 0$

$a - 2b + 3c = 0 \quad 2a - b - c = 0$

$\frac{a}{5} = \frac{+b}{+7} = \frac{c}{3}$

$5x + 7y + 3z - 5 + 7 + 3 = 0$

$5x + 7y + 3z + 5 = 0$

$p = \left|\frac{5+21-21+5}{\sqrt{5^2+7^2+3^2}}\right| = \frac{10}{\sqrt{83}}$

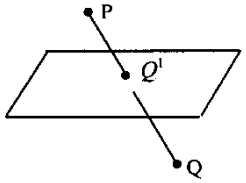
83. If the image of the point P(1, –2, 3) in the plane, 2x + 3y – 4z + 22 = 0 measured parallel to the line, $\frac{x}{1} = \frac{y}{4} = \frac{z}{5}$ is Q, then PQ is equal to :

(1) $2\sqrt{42}$ (2) $\sqrt{42}$

(3) $6\sqrt{5}$ (4) $2\sqrt{5}$

Ans. (3)

$\frac{x-1}{1} = \frac{y+2}{4} = \frac{z-3}{5} = \lambda$



$(1 + \lambda, -2 + 4\lambda, 3 + 5\lambda)$

Satisfy in plane

$2 + 2\lambda - 6 + 12\lambda - 12 - 20\lambda + 22 = 0$

$-6\lambda + 6 = 0 \;\; \therefore \lambda = 1$

$\therefore$ Q¹ (2, 2, 8)

$PQ^1 = \sqrt{1^2 + 4^2 + 5^2}$

$= \sqrt{42} \;\; \therefore PQ = 2\sqrt{42}$

84. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$

Let $\vec{c}$ be a vector such that $\left|\left(\vec{a} \times \vec{b}\right) \times \vec{c}\right| = 3$

and the angle between $\vec{c}$ and $\vec{a} \times \vec{b}$ be 30°. Then $\vec{a} \cdot \vec{c}$ is equal to :

(1) 2 (2) 5

(3) $\frac{1}{8}$ (4) $\frac{25}{8}$

Ans. (3)

$\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k} \quad \hat{b} = \hat{i} + \hat{j}$

$|\vec{c} - \vec{a}| = 3 \quad |(\vec{a} \times \vec{b}) \times \vec{c}| = 3$

$|(\vec{a} \times \vec{b})|\,|\vec{c}|\sin\theta = 3$

where

$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix}$

$|\vec{a} \times \vec{b}| = |2\hat{i} + 2\hat{j} + \hat{k}| = 3$

$|\vec{c}| = \frac{3}{|\vec{a} \times \vec{b}|\sin 30^\circ} = \frac{3}{3 \times \frac{1}{2}} = 2$

$|\vec{c}|^2 + |\vec{a}|^2 - 2\vec{a}.\vec{c} = 9$

$2(\vec{a}.\vec{c}) = 4 + 9 - 9 = 4$

$\vec{a}.\vec{c} = 2$

85. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn is:

(1) 6 (2) 4

(3) $\frac{6}{25}$ (4) $\frac{12}{5}$

Ans. (2)

$n = 10, P = \frac{15}{25} = \frac{3}{5}, q = \frac{2}{5}$

Variance = $npq = 10 \times \frac{3}{5} \times \frac{2}{5} = \frac{12}{5}$

86. For, three events A, B and C, P (Exactly one of A or B occurs)

= P(Exactly one of B or C occurs)

= P(Exactly one of C or A occurs)

= $\frac{1}{4}$ and = P(All the tree events occurs simultaneously) = $\frac{1}{16}$.

Then the probability that at least one of the events occurs, is :

(1) $\frac{7}{16}$ (2) $\frac{7}{64}$

(3) $\frac{3}{16}$ (4) $\frac{7}{32}$

Ans. (3)

$P(A \cup B) - P(A \cap B)$

$= \frac{1}{4} P(B \cup C) - P(B \cap C) = \frac{1}{4}$

$P(A \cap C) - P(A \cap C)$

$= \frac{1}{4} P(A \cap B \cap C) = \frac{1}{16}$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$= \frac{3}{8} + \frac{1}{16} = \frac{7}{16}$

($\because$ $2\{P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A)\} = \frac{3}{4}$)

87. If two different numbers are taken from the set {0, 1, 2, 3 ....., 10}: then the probability that their sum as well as absolute difference are both multiple of 4,

(1) $\frac{12}{55}$ (2) $\frac{14}{45}$

(3) $\frac{7}{55}$ (4) $\frac{6}{55}$

Ans. (2)

n(E) = 6

$\therefore$ (0, 4), (0, 8), (2, 6), (2, 10), (4, 8), (6, 10)

probability = $\frac{6}{{}^{11}C_2} = \frac{6}{55}$

88. If $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$, then the value of cos 4x is :

(1) $\frac{1}{3}$ (2) $\frac{2}{9}$

(3) $-\frac{7}{9}$ (4) $-\frac{3}{5}$

Ans. (1)

$5(\tan^2 x - \cos^2 x) = 2 \cos 2x + 9$

$5\left(\frac{\sin^2 x}{\cos^2 x} - \cos^2 x\right)$

$= 2(\cos^2 x - \sin^2 x) + 9$

$5\left(\frac{1 - \cos^2 x}{\cos^2 x} - \cos^2 x\right)$

$= 2(\cos^2 x - (1 - \cos^2 x)) + 9$

$\Rightarrow 9 \cos^4 x + 12 \cos^2 x - 5 = 0$

$\cos^2 x = \frac{1}{3}$ or $\cos^2 x = \frac{5}{3}$ (not possible)

$\Rightarrow \sin^2 x = \frac{2}{3}$

$\cos 4x = 2 \cos^2 2x - 1$

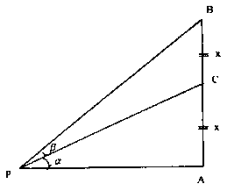
$= 2\left(\frac{1}{9}\right) - 1 = \frac{2}{9} - 1 = \frac{-7}{9}$

89. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that AP = 2AB. If $\angle BPC = \beta$, then $\tan\beta$ is equal to :

(1) $\frac{1}{4}$ (2) $\frac{2}{9}$

(3) $\frac{4}{9}$ (4) $\frac{6}{7}$

Ans. (4)



AP = 2(AB)

AP = 2(2x) = 4x

$\tan\alpha = \frac{1}{4}; \tan(\alpha+\beta) = \frac{2x}{4x} = \frac{1}{2}$

$\frac{\frac{1}{4}+y}{1-\frac{1}{4}y} = \frac{1}{2} \quad 2\left(\frac{1}{4}+y\right) = 1-\frac{y}{4}$

$y\frac{9}{4} = \frac{1}{2}$

$y = \frac{2}{9}$

90. The following statement

$(p \to q) \to [(\sim p \to q) \to q]$ is :

(1) equivalent to $\sim p \to q$

(2) equivalent to $p \to \sim q$

(3) a fallacy

(4) a tautology

Ans. (1)

p	~p	q	~q	p→q	~p→q	(~p→q)→q	(p→q)→ [(~p→q)→q]
T	F	T	F	T	T	T	T
T	F	F	T	F	T	F	T
F	T	T	F	T	T	T	T
F	T	F	T	T	F	T	T